# Appendix, Systematic Modelling

## Phases and componentes

We analyze what are the phases and components that are going to be considered in the equations as in 1



**Phase Component Extensive Intensive**

**property property**



Solid

(

*s*

)

material

(

*m*

)

*M*

*m*

*s*

(

*t*

)

*ρ*

*m*



Table 1: Intensive properties associated with the mass of the components by phases for the heat transport model.

## Material Derivative

The material derivative is present in many developments of the continuous systems, and therefore the following notation will be adopted (D´ıaz-Viera & Ortiz-Tapia, 2018)

*D ∂*

≡ + **v** · ∇ (11)

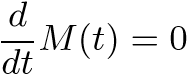


*Dt ∂t*

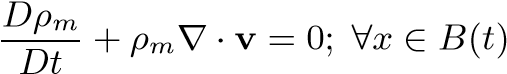
and the *D/Dt* operator will be called the Eulerian representation of the material derivative.

## Mass balance

Because there is mass conservation (only solid objects are studied) (D´ıaz-Viera & Ortiz-Tapia, 2018)

 (12)

and it is obtained the continuity equation

 (13)

Linear momentum balance APPENDIX, SYSTEMATIC MODELLING



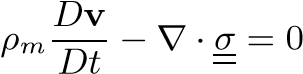
## Linear momentum balance

Because we have mass conservation (Eq. 13), it is obtained the first Law of Cauchy for the mechanics of continuum media: (D´ıaz-Viera & Ortiz-Tapia, 2018)

*D***v**

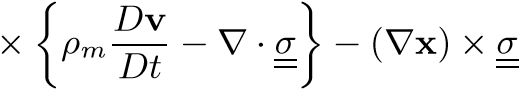
**b** (14)

since it is assumed that linear momentum is conserved:

 (15)

## Angular momentum balance

Because there is mass conservation (Eq.13), the local balance of angular momentum equation can be written as (D´ıaz-Viera & Ortiz-Tapia, 2018)

**x**= 0 (16)

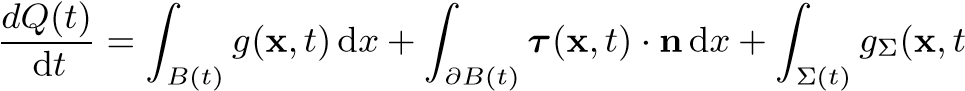
but since= 0 because of the conservation of linear momentum:

 (17)

So we have conservation of angular momentum.

## Energy balance

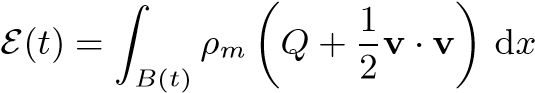
Let the general global balance equation be (D´ıaz-Viera & Ortiz-Tapia, 2018)

)d*x* (18)

where

* *g*(*x,t*) is that which is generated or destroyed inside the body *B*(*t*).
* τ(**x***,t*) is that which enters or outs the boundary of the body *∂B*(*t*)
* *g*Σ(**x***,t*) is that which is generated or destruyed inside the discontinuity Σ(*t*)

Let the global balance of energy be (D´ıaz-Viera & Ortiz-Tapia, 2018)

 (19)

So we have as an extensive property, the energy E(*t*).

And as an intensive property: , where *Q* is the internal energy per unit of mass.

if we have that: *g* ≡ *ρm*(*q*0 + **b** · **v**), and **v**, where

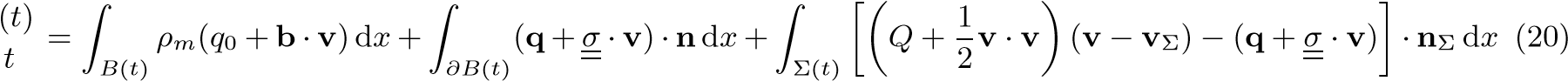
* *ρmq*0 is the heat source, per unit volume.
* *ρm***b** · **v** is the work per unit of time and per unit volume realized by the forces inside the body. • **q** · **n** is the heat flux through the boundary per unit time, per unit of area,

 **v** is the work per unit of time per unit of area realized by the traction forces

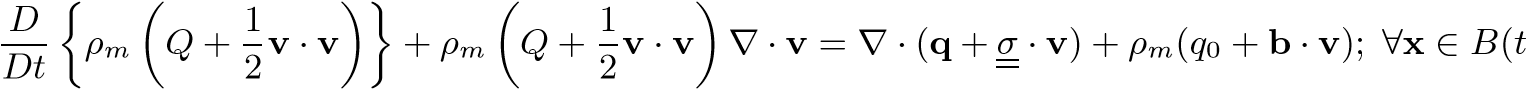
* **v**Σ the velocity at the discontinuity.

Then,



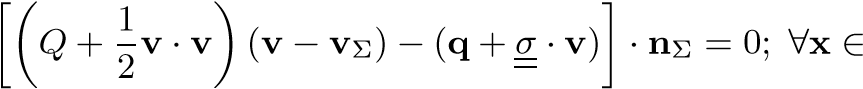
d

The local balance equation is

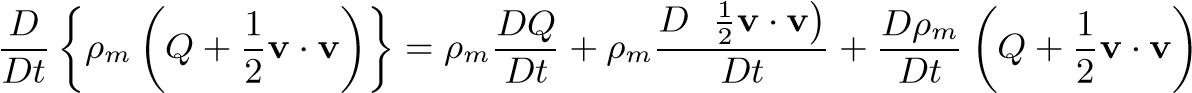
) (21)

Constitutive Laws APPENDIX, SYSTEMATIC MODELLING

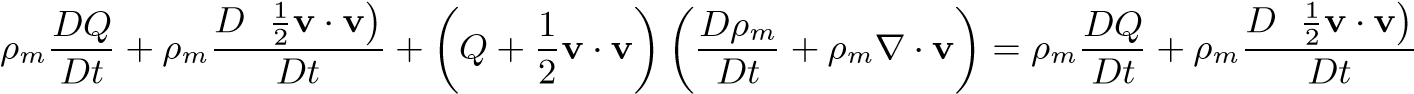
The jump conditions within a discontinuity (if any) are

Σ (22)

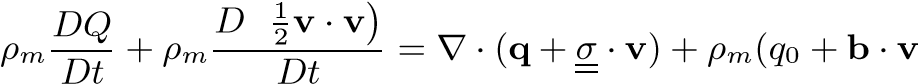
Developing the material derivative term, it results in

 (23)

Substituting Eq.23 into the left member of Eq.21 it is obtained

 (24)

since *DρDtm* + *ρm*∇ · **v** = 0 because of mass conservation, Eq.21 becomes

) (25)

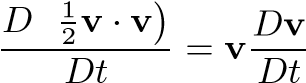
If one takes in account that

**v** (26)

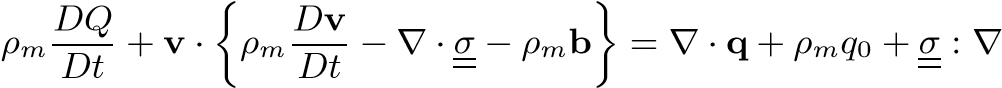




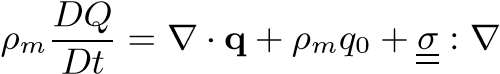
and that

 (27)

then, substituting Eqs.26 and 27 into Eq.25 it is obtained:

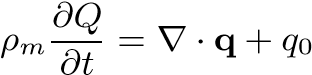
**v** (28)

Bearing in mind that= 0 because of the conservation of linear momentum, one finally obtains the general equation of balance of energy, under the assumptions made so far of conservation of mass, linear and angular momentum.

**v** (29)

where. We can assume further that heat (energy) acts only upon solid materials ( no

tractions forces), so **v** = 0, and by the definition of the material derivative (Eq.11), one obtains

 (30)

## Constitutive Laws

**Fourier’s Law (conduction)** The differential form of Fourier’s law of thermal conduction shows that the local heat flux density, **q**, is equal to the product of thermal conductivity, *k*, and the negative local temperature gradient, −∇*T*. The heat flux density is the amount of energy that flows through a unit area per unit time

**q** = −*λm*∇*T* (31)

where (including the SI units)

* *T* = *T*(*x,y,z,t*) (in general) is the local temperature
* **q** is local heat flux density, [W · m−2]
* *λm* is the material’s conductivity, [W · m−1 · K−1] ,

The thermal conductivity, *k*, is often treated as a constant, though this is not always true. While the thermal conductivity of a material generally varies with temperature, the variation can be small over a significant range of temperatures for some common materials. In anisotropic materials, the thermal conductivity typically varies with orientation; in this case *k* is represented by a second-order tensor. In non-uniform materials, *k* varies with spatial location. (Bergman, Incropera, Lavine, & Dewitt, 2011)

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The dimensional analysis of the right-hand-side (rhs) of Eq.31 renders

[W · m−1 ·✟*K*−✟1] · [✚K] = [W · m−2] (32)

as expected.

**Newton’s cooling Law (convection)** Newton’s law of cooling states that the rate of heat loss of a body is directly proportional to the difference in the temperatures between the body and its surroundings provided the temperature difference is small and the nature of radiating surface remains the same. As such, it is equivalent to a statement that the heat transfer coefficient, which mediates between heat losses and temperature differences, is a constant. This condition is generally true in thermal conduction (where it is guaranteed by Fourier’s law), but it is often only approximately true in conditions of convective heat transfer, where a number of physical processes make effective heat transfer coefficients somewhat dependent on temperature differences. So, the convection differential equation is

(Bergman et al., 2011) *dQ*

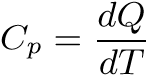
 = *h* · (*T*(*x,y,z,t*) − *T env*) (33)

d*t*

where

* *T* = *T*(*x,y,z,t*) (in general) is the local temperature
* *hconv* is the average convective heat transfer coefficient [W · m−2K−1]
* *Tenv* is the environment temperature [ K ]

From the definition of heat capacity comes the relation

*.* (34)

Differentiating this equation with regard to time gives the identity (valid so long as temperatures in the object are uniform at any given time):

*dQ dT*



*t*

=

*C*

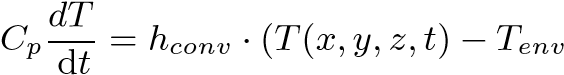
*p*



d

*t*

*.* (35) d

Where *Cp* is the material heat capacity at constant pressure [J*/*(kg · K)] (Cannas et al., 2012); therefore Eq.33 can be written as 

) (36)

|  |  |
| --- | --- |
| The dimensional analysis of the lhs of Eq.36 renders |  |
| [J · (kg−1 ·✟*K*−✟1)] · [✚*K* · s−1] = [W · kg−1]  upon multiplying by a density *ρ*[kg · m−3], renders | (37) |
| [*kg* · m−3] · [W ·✟*kg*✟−✟1] = [W · m−3]  The dimensional analysis of the rhs of Eq.36 renders | (38) |
| ✟  [W · m−2✟*K*−1] · [✚*K*] = [W · m−2] | (39) |

**Stephan-Boltzmann law (radiation)** The Stefan-Boltzmann law describes the total power, per surface area *P* radiated from a surface in terms of the difference in temperature of a radiating object with its surroundings (Bergman et al., 2011; Cannas et al., 2012):

*P* = *σε*(*Tenv*4 − *T*4) (40)

where

* *P* is the total power radiated, per surface area [W · m−2]
* *T* = *T*(*x,y,z,t*) (in general) is the local temperature
* *Tenv* is the environment temperature [ K ]
* *σ* is the Stefan-Boltzmann constant with value 5*.*670373 × 10−8 [W · m−2 · K−4]
* *εm* is the surface emissivity, which depends on the material and has a value between 0 and 1, that is with dimensions [1].

References References

The dimensional analysis of Eq.40 renders

[W · m−2] = [W · m−2 ·✟*K*−✟4] · [1] · [✟*K*−✟4] (41)

as expected.