Research Article

Random oscillations of nonlinear systems with distributed Parameter

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Abstract

The article analyzes random vibrations of nonlinear mechanical systems with distributed parameters. The motion of such systems is described by nonlinear partial differential equations with corresponding initial and boundary conditions. In our case, the system as a whole is limited, so any motion can be considered as the sum of the natural oscillations of the system, i.e. in the form of an expansion of the boundary value problem in terms of own functions. The use of the theory of random processes in the calculation of mechanical systems is a prerequisite for the creation of sound design methods and the creation of effective vibration protection devices, these methods allow us to investigate dynamic processes, to determine the probabilistic characteristics of displacements of points of the system and their first two derivatives. In the work established these conditions are met, they provide effective vibration protection of the system under study with wide changes in the pass band of the frequencies of the random vibration effect, and the frequency of the disturbing force is much greater than the natural frequency of the system as a whole, in addition, with an increase in the damping capacity of the elastic-damping link of the system, the intensity of the random process significantly decreases, which in turn leads to a sharp decrease in the dynamic coefficient of the system.

Introduction

The work analyzes random vibrations of nonlinear mechanical systems with distributed parameters, in particular, a long rod with a vibration protection device. A rod with a vibration protection device is a complex mechanical system characterized by inertial, elastic, viscous and dissipative properties, while its parameters are continuously distributed in space. The motion of such systems is described by nonlinear partial differential equations with corresponding initial and boundary conditions. In our case, the system as a whole is limited, so any motion can be considered as the sum of the natural oscillations of the system, i.e. in the form of an expansion of the boundary value problem in terms of own functions. The use of the theory of random processes in the calculation of mechanical systems is a prerequisite for the creation of sound design methods and the creation of effective vibration protection devices, these methods allow us to investigate dynamic processes, to determine the probabilistic characteristics of displacements of points of the system and their first two derivatives. It is known from the theory of random vibrations that forced vibrations that arise in vibration systems under stationary influences are a stationary random process. The analysis of random oscillations is reduced to the determination of their probabilistic characteristics according to the probabilistic characteristics of the vibration effect [1,2].

We have experimentally established that the elastic–damping elements in the vibration protection device under cyclic loading of compression–unloading have a hysteresis characteristic of the parabolic type, the mathematical described of which is proposed in the first and have the form:

$$\sigma(x,t) = \frac{1}{2}\left[A_n\left(k_{cp}E\right)\left(\partial u / \partial x\right)^n \left(\mathrm{sgn}\, \partial^2 u / \partial x \partial t + 1\right) + A_m\left(k_{cp}E\right)\left(\partial u / \partial x\right)^m \left(\mathrm{sgn}\, \partial^2 u / \partial x \partial t - 1\right)\right]$$

(1)

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Here $u(x,t)$ - displacement along the X-axis of an arbitrary cross-section of the bar; A_n, A_m -

coefficients depending on the elastic modulus E and the stiffness of the elastic-damping element k_{cp}; – n and m exponents, depending on the damping quality of the vibration protection device.

The approximated shapes of the parabolic hysteresis loop of the damping elements in the static and dynamic compression-unloading cycle are described by the expressions respectively.

$$\phi\left(u,u^{\bullet}\right)=k_{cp}u\left(1+\lambda\,\mathrm{sgn}\,\partial u\,/\,\partial t\right),\ \phi^{\bullet}\left(u,u^{\bullet}\right)=k_D\left(u+u_D^0\right)\left(1+\lambda\,\mathrm{sgn}\,\partial\left(u+u_D^0\right)/\,\partial t\right),$$

The differential equation of motion of the system under study takes the form

$$m\partial^2 u\,/\,\partial t^2 - c^2\partial^2 u\,/\,\partial x^2 + \mu\,/\,\rho A\partial u\,/\,\partial t +$$
$$+k_D\left(u+u_D^0\right)\left(1+\lambda\,\mathrm{sgn}\,\partial\left(u+u_D^0\right)/\,\partial t\right)=F\left(u,t\right) \tag{2}$$

Here m – is the mass of the rod per unit length, μ – is the coefficient of viscous resistance of the elastic-damping link of the vibration protection device, k_D – the average static stiffness of the elastic-damping element, $c^2 = E\rho^{-1}$, E– Young's modulus of the rod, ρ – mass density, A– is the cross-sectional area of the elastic-damping, $F(u,t)$ – random disturbing force, λ – is an indicator of the scattering ability of the energy of forced vibrations of the elastic-damping element, $u(x,t)$ – displacement of the studied system as a whole along the X-axis.

One of the most effective methods for solving equation (2) is the method of approximate reduction to a system with a finite number of degrees of freedom using expansions in suitable basis functions, i.e. according to the proper forms of the corresponding linear system [3].

The boundary and initial conditions for the $u(x,t)$ system in our case have the form:

$$u\big|_{t=0} = 0,\ \partial u\,/\,\partial t\big|_{t=0} = 0,\ \partial\left[u\left(0,t\right)\right]/\,\partial x = F_0 e^{i\omega t},\ \partial\left[u\big|l,t\big|\right]/\,\partial x = 0 \tag{3}$$

Methods

The solution to differential equation (2) satisfying (3) will be sought in the form, here – l is the length of the system as a whole.

$$u\left(x,t\right)=\Sigma_{i=1}^N u_i\left(t\right)\sin\pi ix. \tag{4}$$

Formulation of (4) into equation (2) and the corresponding standard calculations lead to a system of outlined nonlinear differential equations

$$m_1\ddot{x}_1 + K_1 x_1 + \mu\dot{x}_1 - K_2^D\left[\left(x_2-x_1\right)+\left(x_2-x_1\right)_D\right]\times\left\{1+\lambda\,\mathrm{sgn}\,\partial\left[\left(x_2-x_1\right)+\left(x_2-x_1\right)_D\right]/\,\partial t\right\}=0$$

$$m_2\ddot{x}_2 + K_2^D\left[\left(x_2-x_1\right)+\left(x_2-x_1\right)_D\right]\times\left\{1+\lambda\,\mathrm{sgn}\,\partial\left[\left(x_2-x_1\right)+\left(x_2-x_1\right)_D\right]/\,\partial t\right\}=F\left(t\right). \tag{5}$$

Here m_1– is the mass of the long rod as a whole, m_2 – the total mass of elastic elements, K_1 - equivalent stiffness of elastic-damping elements, K_2^D - average dynamic stiffness of elastic elements, $\left(x_2-x_1\right)_D$ – residual movement of elastic-damping elements during a dynamic compression-unloading cycle, μ – coefficient of viscous resistance of the system as a whole, x_1, x_2 - generalized coordinates of the displacement of system elements in the longitudinal directions, $F(t)$ – is a centered normal random process.

To determine the probabilistic characteristics and use the method of static linearization. Obviously, all probabilistic characteristics of a normal random process are fully determined if the mathematical expectation and correlation function of the investigated random process are known.

The nonlinear function that determines the damping of the energy of forced vibrations in a dynamic compression-unloading cycle in this case has the form:

$$z\left(t\right)=\left(x_2-x_1\right)+\left(x_2-x_1\right)_D;\quad \dot{z}\left(t\right)=\left(\dot{x}_2-\dot{x}_1\right)+\left(\dot{x}_2-\dot{x}_1\right)_D; \tag{6}$$

$$f\left[z(t),\dot{z}(t)\right]=K_2^D\left[\left(x_2-x_1\right)+\left(x_2-x_1\right)_D\right]\times\left\{1+\lambda\,\text{sgn}\,\partial\left[\left(\dot{x}_2-\dot{x}_1\right)+\left(\dot{x}_2-\dot{x}_1\right)_D\right]\right\}/\partial t,$$

it is necessary to know the laws of distribution of random processes $z(t), Z^*(t)$.

We assume that they also have a normal distribution.

The method of statistical linearization is reduced to replacing the nonlinear function (6) with a linear one in this form:

$$f^*(t)=\phi_0+\alpha z^0+\beta\dot{z}_0;\quad z^0=z(t)-m_z;\quad \dot{z}^0(t)=\dot{z}(t).$$

The coefficients, α and β can be determined from the condition of the minimum variance of the investigated random process

$$\Delta f(t)=f(t)-f^*(t).$$

It is known that the variance of a stationary random process does not depend on time; therefore, the variance of the process under study can be determined as follows

$$M\left[\Delta f(t)^2\right]=\int\limits_{-\infty}^{+\infty}\int\left[f(u,v)-\phi_0-\alpha\left(u-m_u\right)-\beta v\right]z^2\omega_2(u,v)\,dudv$$

Where $\omega_2(u,v)$ is the joint probability distribution density of the processes $z(t)$ and $\dot{z}(t)$,

m_u – is the mathematical expectation of a random process x_2-x_1.

Which characterizes the behavior of the process in time on average δ_v^2 – process variance

Characterizing the dispersion of the values of a random function in time relative to the average value $\dot{x}_2-\dot{x}_1$ is the variance of the process.

It is taken into account that for a stationary process, the average value of the velocity difference is equal to zero

$$m_{x2}-m_{x1}=0$$

Minimization of functional (7) makes it possible to determine the linearization coefficients

$$\phi_0=\frac{\left(\delta_u\cdot\delta_v\right)^1}{2\pi}\int$$

$$\phi_0=\frac{\left(\delta_u\cdot\delta_v\right)^1}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}K_2^{1)}\left(u-u_{1)}\right)\left(1+\lambda\,\text{sgn}\,v\right)\exp\left[\frac{\left(u-m_u\right)^2}{2\delta_u^2}-\frac{v^2}{2\delta_v^2}\right]dudv;$$

$$\alpha=\frac{\sigma_u^{-3}\cdot\delta_v^{-1}}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}K_2\left(1\pm\lambda\,\text{sgn}\,v\right)\exp\left[-\frac{\left(u-m_u\right)^2}{2\delta_u^2}-\frac{v^2}{2\delta_v^2}\right]\times$$

$$\times\left(u-m_u\right)dudv;$$

$$\beta=\frac{\sigma_u^{-1}\cdot\delta_v^{-3}}{2\pi}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}K_2\left(1\pm\lambda\,\text{sgn}\,v\right)\exp\left[\frac{\left(u-m_u\right)^2}{2\delta_u^2}-\frac{v^2}{2\delta_v^2}\right]v\,dudv.$$

(7)

Let's go to polar coordinates and get

$$\frac{u-m_u}{\sigma_u}=r\cdot\cos\Theta;\quad \frac{v}{\sigma_v}-r\cdot\sin\Theta;\quad r\geq0;$$

$$u=\sigma_u r\cos\Theta+m_u;\quad v=\sigma_v r\sin\Theta.$$

Jacobian

$$I = \begin{vmatrix} \dfrac{\partial u}{\partial r} & \dfrac{\partial u}{\partial \Theta} \\[2mm] \dfrac{\partial v}{\partial r} & \dfrac{\partial v}{\partial \Theta} \end{vmatrix} = \begin{vmatrix} \sigma_u \cos\Theta & \sigma_u\, r \sin\Theta \\ -\sigma_v \cos\Theta & \sigma_v\, r \cos\Theta \end{vmatrix} = r\sigma_u \sigma_v .$$

After substitution and corresponding transformations, we get

$$\phi_0 = \frac{K_2}{2\pi}\int_0^{2\pi} d\Theta \int_0^{\infty} e^{-\frac{r^2}{2}}\left(\sigma_u r \cos\Theta + m_u\right)\left[1 + \lambda\, \mathrm{sgn}\left(\sigma_v r \sin\Theta\right)\right] dr;$$

$$\alpha = \frac{K_2}{2\pi\sigma_u^2}\int_0^{2\pi} d\Theta \int_0^{\infty} e^{-\frac{r^2}{2}} r^3 \left(\sigma_u r \cos\Theta + m_u\right)\left[1 + \lambda\, \mathrm{sgn}\left(\sigma_v r \times \sin\Theta\right)\right]\sigma_u \cos\Theta\, dr;$$

$$\beta = \frac{K_2}{2\pi\sigma_v^2}\int_0^{2\pi} d\Theta \int_0^{\infty} e^{-\frac{r^2}{2}} r^3 \left(\sigma_u r \cos\Theta + m_u\right)\left[1 + \lambda\, \mathrm{sgn}\left(\sigma_v r \times \cos\Theta\right)\right]\sigma_v \sin\Theta\, dr.$$

Integration, we get

$$\phi_0 = K_2 m_u; \quad \alpha = K_2;$$
$$\beta = -\sqrt{\frac{2}{\pi}}\lambda K_2 m_u \delta_v^{-1}\, \mathrm{sgn}\,\delta_v .$$

System (5) of randomized nonlinear differential equations after statistical linearization takes the form

$$\begin{cases} m_1 \ddot{x}_1^0 + K_1 x_1^0 + \mu x_1^0 - K_2 m_{x_2 - x_1} - K_2\left(x_2^0 - x_1^0\right) + \\[2mm] \quad + \lambda K_2 m_{x_2 - x_1}\sqrt{\dfrac{2}{\pi}}\left(\dot{x}_2^0 - \dot{x}_1^0\right)\dfrac{\mathrm{sgn}\,\delta\,\dot{x}_2^0 - \dot{x}_1^0}{\delta_{\dot{x}_2 - \dot{x}_1}} = 0 \\[4mm] m_2 \ddot{x}_2^0 + K_2 m_{x_2 - x_1} + K_2\left(x_2^0 - x_1^0\right) - \lambda K_2 m_{x_2 - x_1} \times \\[2mm] \quad \times \sqrt{\dfrac{2}{\pi}}\left(\dot{x}_2^0 - \dot{x}_1^0\right)\dfrac{\mathrm{sgn}\,\delta\,\dot{x}_2^0 - \dot{x}_1^0}{\delta_{\dot{x}_2 - \dot{x}_1}} = F(t) \end{cases};$$

(8)

$$\dot{x}_1^0 = \dot{x}_1(t); \quad x_1^0 = x_1(t) - m_{x_1};$$
$$\dot{x}_2^0 = \dot{x}_2(t); \quad x_2^0 = x_2(t) - m_{x_2}.$$

Let us determine the spectral density and mathematical expectation both by the spectral densities and the mathematical expectation of the vibration action $F(t)$. Suppose, where is the normal white noise, the mathematical expectation of the process $F(t)$. All realizations of random processes and are continuous and differentiable at least twice. Taking into account the above, from (8) we obtain

$$m_1 p_2 + (\beta + \mu) p + (K_1 + \alpha) x_1^0 = (\beta p + \alpha) x_2^0;$$
$$(m_2 p_2 + \beta p + \alpha) x_2^0 - (\beta p + \alpha) x_1^0 = F_0;$$

(9)

Where $P = d/dt$ is the differentiation operator?

Similarly, expressions for determining mathematical expectations and have the form

$$\left(K_1 + \alpha\right) m_{x_1} - \alpha m_{x_2} = 0 \, ;$$

$$\alpha\left(m_{x_1} - m_{x_2}\right) = m_v \, .$$

$$m_{x_1} = \frac{m_v}{K_1}; \quad m_{x_2} = \frac{\left(K_1 + K_2\right)}{K_1 K_2} \cdot m_v; \quad m_{x_2 - x_1} = m_{x_2} - m_{x_1} = \frac{m_v}{K_2} \, ;$$

Let us assume that are the spectral densities of the processes and, respectively.

From (9) we obtain

$$x_1^0 = \frac{F^0\left(\beta p + \alpha\right)}{\left[m_1 p^2 + \left(\beta + \mu\right) p + K_1 + \alpha\right]\left[m_2 p^2 + \left(\beta + \alpha\right) - \left(\beta p + \alpha\right)^2\right]}$$

$$x_1^0 = \frac{F_0\left[m_1 p_2 + \left(\beta + \mu\right) p + K_1 + \alpha\right]}{m_1 m_2 p_4 + \Theta_1 p_3 + \left(K_1 \beta + \alpha \mu\right) p + \alpha K_1} \, ;$$

$$\sigma_{x_1}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{F_0 \left|\beta\left(j\omega\right) + \alpha\right|^2 d\omega}{\Theta^3\left(j\omega\right)} \, ;$$

$$\sigma_{x_2}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{F_0 \left|m_1\left(j\omega\right)^2 + \left(\beta + \mu\right) j\omega + K_1 + \alpha\right|^2 d\omega}{\Theta_3\left(j\omega\right)} \, ;$$

$$\Theta_3 = \left|m_1 m_2\left(j\omega\right)^4 + \Theta_1\left(j\omega\right)^3 + \Theta_2\left(j\omega\right)^2 + \left(K_1 \beta + \alpha \mu\right)\left(j\omega\right) + \alpha K_1\right|^2 \, ;$$

$$\Theta_1 = m_1 \beta + m_2\left(\beta + \mu\right); \quad \Theta_2 = m_2\left(K_1 + \alpha\right) + \beta\left(\beta + \mu\right).$$

$$S_{F_0}\left(\omega\right) = F_0 \left(-\infty < \omega < \infty\right).$$

After integration, we find

$$\sigma_{x_1}^2 = \frac{F_0 \left\{\Theta_4 \beta^2 + \alpha K_1^{-1}\left[m_1 m_2\left(K_1 \beta - \alpha \mu\right) - \Theta_4 \Theta_5\right]\right\}}{2\left[m_1 m_2\left(K_1 \beta - \alpha \mu\right) - \Theta_4^2 \alpha \cdot K_1 - \Theta_4 \Theta_5\left(K_1 \beta - \alpha \mu\right)\right]} \, ;$$

$$\sigma_{x_2}^2 = \frac{F_0 \left\{\Theta_4\left(\mu + \beta\right)^2 + \left(K_1 - \alpha\right)^2\left(\alpha K_1\right)^{-1}\left[m_1 m_2\left(K_1 \beta - \alpha \mu\right) - \Theta_4 \Theta_5 - m_1^2\left(K_1 \beta - \alpha \mu\right)\right]\right\}}{2\left[m_1 m_2\left(K_1 \beta - \alpha \mu\right) - \Theta_4^2 \alpha \cdot K_1 - \Theta_4 \Theta_5\left(K_1 \beta - \alpha \mu\right)\right]} \, ;$$

$$\Theta_1 = m_1 \beta + m_2\left(\beta + \mu\right); \quad \Theta_2 = m_2\left(K_1 + \alpha\right) + \beta\left(\beta + \mu\right)$$

$$\Theta_3 = \left|m_1 m_2\left(j\omega\right)^4 + \Theta_1\left(j\omega\right)^3 + \Theta_2\left(j\omega\right)^2 + \left(K_1 \beta + \alpha \mu\right)\Theta_1\left(j\omega\right) + \alpha K_1\right|^2$$

$$\Theta_5 = m_2\left(K_1 + \alpha\right) + \beta\left(\beta - \mu\right).$$

$$\Theta_4 = m_1 \beta + m_2\left(\beta - \mu\right).$$

From the analysis of the last two expressions, we find that the root-mean-square values (displacements, deformations) of the system under study as a whole are in direct proportion to the value of the random process and take maximum values at

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$$K_2 = \frac{\pi m_1 (m_2 + 1) \delta^2_{\dot{x}_2 - \dot{x}_1}}{4\lambda^2 (m_1 + m_2) m^2_{x_2 - x_1}}$$

(10)

From (8), it is possible to determine the probabilistic characteristics of the force response of the elastic-damping elements of the system, which make it possible to assess the effectiveness of vibration protection of the studied system as a whole.

the notation

$$u_1 (x_1, \dot{x}_1) = K_1 x_1^0 + \mu \dot{x}_1^0 \; ;$$

$$u_2 (x_1, \dot{x}_2) = K_1 \left(m_u - x_2^0 - \lambda m_u \sqrt{\frac{2}{\pi}} x_2^0 \times \delta_v^{-1} \, \mathrm{sgn} \, \delta_v \right);$$

$$u_3 (x_1, \dot{x}_1) = K_2 \left(x_1^0 - \lambda m_u \sqrt{\frac{2}{\pi}} x_1^0 \times \delta_v^{-1} \, \mathrm{sgn} \, \delta_v \right);$$

$$u (x_1, x_2, \dot{x}_1, \dot{x}_2) = u (x_2, \dot{x}_2) - u_3 (x_1, \dot{x}_1).$$

Substituting these expressions into system (8), we obtain

$$m_1 \ddot{x}_1 + u_1 (x_1, \dot{x}_1) = u (x_2, \dot{x}_2) - u_3 (x_1, \dot{x}_1);$$

$$m_2 \ddot{x}_2 + u_2 (x_2, \dot{x}_2) = F (t) + u_3 (x_1, \dot{x}_1).$$

(11)

We assume that random functions and are not correlated, i.e. correlation function is zero

$$K_{u_2 u_3} (\tau) = 0, \; \tau = t - t'.$$

From (11) we determine the spectral density of the function

$$\sigma_u^2 = (2\pi) \int_{-\infty}^{\infty} S_u (\omega) d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{m_2 \omega^2 (2\alpha - m_2 \omega^2) S_F (\omega) d\omega}{(\alpha - m_2 \omega^2)^2 - \beta^2 \omega^2}.$$

$$S_u = S_{u_2} - S_{u_3} = \frac{m_2 \omega^2 (2\alpha - m_2 \omega^2)}{(\alpha - m_2 \omega^2)^2 - \beta^2 \omega^2} S_F (\omega)$$

(12)

Since the investigated random process is broadband and close to white noise, therefore, the spectral density of the random vibration action *F(t)* – is determined by the following expression

$$S_F (\omega) = \frac{2\mu\xi \cdot \sigma_F^2}{\left| \mu - \omega^2 + \xi j\omega \right|^2}.$$

(13)

The parameter η - is approximately equal to the square of the prevailing frequency of the random action *F(t)*, respectively, the parameter ξ determines the bandwidth of the random vibrations of the elastic-damping link of the system under study, as a mechanical filter.

Substituting (12) into (13), we obtain

$$\sigma_u^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2\mu\xi \cdot \sigma_F^2}{\left| \alpha - m_2 \omega^2 + \beta j\omega \right|^2 \left| \mu - \omega^2 + \xi j\omega \right|^2},$$

Where is the root-mean-square value of the force vibration effect?

Is the root-mean-square value of the force reaction of the elastic-damping link of the system under study? Accordingly, the

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square of the dynamic coefficient of the system is determined by the expression

$$K^2 = \frac{\sigma_u^2}{\sigma_F^2} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\left(2m_2\omega^2\alpha - m_2^2\omega^4\right)\mu\xi \cdot d\omega}{\left|\alpha - m_2\omega^2 + \beta j\omega\right|^2 \left|\mu - \omega^2 + \xi j\omega\right|^2} .$$

We got an integral of the form

$$I_4 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{g_4\left(j\omega\right)}{h_4\left(j\omega\right) \cdot h_4\left(-j\omega\right)} d\omega ,$$

Where and are fourth degree polynomials

$$h_4\left(j\omega\right) = m_2\left(j\omega\right)^4 + \left(m_2\xi\right)\left(j\omega\right)^3 + \left(m_2\mu + \beta\xi + \alpha\right)\left(j\omega\right)^2 +$$
$$+ \left(\beta\mu + \alpha\xi\right)\left(j\omega\right) + \alpha\mu ;$$
$$g_4\left(\Delta\omega\right) = -2\mu\xi m_2^2\left(j\omega\right)^4 - 4\alpha\mu\xi m_2\left(j\omega\right)^2 ;$$
$$h_4\left(-j\omega\right) - h_4\left(j\omega\right) .$$

By integrating, we get

$$K^2 = \frac{m_2\mu\left[\left(m_2\xi - K_2\lambda r\right) + \lambda r\left(\alpha - m_2 r\right)\right]}{\lambda r\left[\left(K_2 - m_2 r\right)^2 + \left(m_2\xi - \lambda r\right)\right] \cdot K_2} \times \frac{1}{\left(\xi - \lambda\mu r\right)} ,$$
$$r = \sqrt{2\pi^{-1}} \, m_4\sigma_v^{-1} \cdot \mathrm{sgn}\,\sigma_v .$$

$$(14)$$

For a variable η, the condition for the critical value of the dynamism coefficient of the system under study is determined by the expression

$$m_2\mu^2 - \left[2\alpha m_2 - \beta\left(m_2\xi + \beta\right)\right]\mu + \left[\alpha^2 + \alpha\xi\left(\beta + m_2\xi\right)\right] = 0 .$$

Conception only has a positive root

$$\mu = \frac{K_2}{m_2} - \left(\frac{\beta + m_2\xi}{m_2^2}\right)\left(\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} - K_2 m_2}\right) .$$

When the transmission width of the forced frequencies of the system under study is small and takes on a value when the dynamic coefficient of the system takes on a maximum value

$$\mu_0 \approx K_2 m_2^{-1} ,$$

$$K_{max}^2 = \frac{\pi}{2} \cdot \frac{\sigma_{\dot{x}_2 - \dot{x}_1}^2}{\lambda^2 m_{\dot{x}_2 - \dot{x}_1}^2 \cdot \mu_{pe}} .$$

Function

$$\phi\left(K_2\right) = K_2^2\left(\lambda^2 r^2\mu - \lambda\xi r + 1\right) + K_2\left[m_2\left(\xi^2 - 2\mu - \lambda r\mu\xi\right)\right] - m_2^2\mu^2 ,$$

At the tested stationary point takes the maximum value

$$K_2^{cm} = \frac{m_2\left(\lambda r\xi\mu - \xi^2\right)}{2\left(\lambda^2 r^2\mu - \lambda r\xi - 1\right)},$$

$$\xi < \left(1 + \lambda^2 r^2\mu\right)\cdot\left(\lambda r\right)^{-1}$$

(15)

Thus, when conditions (15) are satisfied, the dynamic coefficient of the system under study reaches its maximum value. When the dynamic coefficient of the system is less than one, then the use of the elastic–damping link of the system works effectively and accordingly reduces the amplitude of the forced vibrations of the system as a whole.

From (14) we can determine all conditions under which the coefficient of dynamism of the system becomes less than one

$$Q_1 < \mu; \quad Q_2 < \mu;$$

$$m_u > \frac{1}{\lambda}\sqrt{\pi m_2 \sigma_\nu K_2^1\left(\sigma_\nu\left(\sqrt{\sigma_\nu^2 - 0,5}\right)\right)};$$

$$Q_{1,2} = K_2 m_2^{-1} \pm K_2\left(2\pi m_2^2\sigma_\nu^2\right)^{-1}\times\left(\sqrt{2\pi m_2^2\sigma_\nu^4}\right) -$$

$$- K_2\lambda^2 m_u^2\left(4\pi m_2\sigma_\nu^2 - K_2\lambda^2 m_u^2\right) - K_2\lambda^2 m_u^2);$$

$$\sigma_\nu^2 > 0,5.$$

Conclusion

When these conditions are met, they provide effective vibration protection of the system under study with wide changes in the pass band of the frequencies of the random vibration effect, and the frequency of the disturbing force is much greater than the natural frequency of the system as a whole, in addition, with an increase in the damping capacity of the elastic–damping link of the system, the intensity of the random process significantly decreases, which in turn leads to a sharp decrease in the dynamic coefficient of the system.

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